SULTAN QABOOS UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 4141 Fall 2011

Test 2 Time: 75 minutes

Name:. . . . . Section: . . . . . Number. . . . .

Important Instructions

- Make sure you write your name, number and section number on the exam paper and on the solution booklet.
- Solve all questions. Make sure you show your complete, mathematically correct and neatly written solution.
- You are NOT allowed to share calculators or any other material during the test under any circumstances.
- Cellular phones are NOT allowed to be used in class.

Q1: *(3+3+2 points)*

Consider the points $(1,2),(2,1),(3,4),(4,3)$ in answering each of the following:

(i) Use a Lagrange interpolating polynomial of least degree to interpolate the points.

(ii) Use the divided differences method to interpolate the points.

(iii) Is the polynomial you obtain in (i) same as the one you obtain in (ii). Justify your answer.

Q2: Answer each of the following: *(4+2+3 points)*

(i) Use a suitable three-point formula (make sure you write the formula you use) to determine each missing entry in the following table:

x	$f(x)$	$f'(x)$
8.1	16.944	
8.3	17.565	
8.5	18.191	3.140
8.7	18.820	3.164

(ii) If you know that the data in the above table is for $y = x\ln(x)$. What is the actual error in each case?

(iii) Use the error formula to find an error bound when you approximate $f'(8.1)$.

Q3: *(6 points)*

Let $h = \frac{1}{3}(b-a), x_0 = a, x_1 = a+h$ and $x_2 = b$. Find the degree of precision of the quadrature formula

$$\int_a^b f(x)dx = \frac{9}{4}hf(x_1) + \frac{3}{4}hf(x_2).$$

Q4: *(6 points)*

Approximate the integral $\int_0^1 e^{-x^2}dx$ using

(i) the Trapezoidal rule (ii) Simpson's rule.

Q5: *(4+3+3+1 points)*

Consider the nonlinear system

$$x_1^2 - 10x_1 + x_2^2 + 8 = 0$$
$$x_1x_2^2 + x_1 - 10x_2 + 8 = 0.$$

and the set

$$D = \{[x_1, x_2]^t : \ 0 \le x_1 \le 1.5, \ 0 \le x_2 \le 1.5\},$$

then answer each of the following:

(i) Write the system as $x_1 = g_1(x_1, x_2)$ and $x_2 = g_2(x_1, x_2)$ such that $G = [g_1, g_2]^t$ maps the set D into itself.

(ii) Show that for some $k < 1$, we have

$$\left|\frac{\partial g_i(x_1, x_2)}{\partial x_j}\right| \le \frac{k}{2}$$

for each $j = 1, 2$ and each $i = 1, 2$.

(iii) Start with $X_0 = [0, 1]^t$ and use $X_{k+1} = G(X_k)$ to find X_1 and X_2.

(iv) Evaluate $||X_1 - X_0||_\infty$.

Good Luck