

SULTAN QABOOS UNIVERSITY  
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 4141

Fall 2011

Test I

Time: 70 minutes

Name: . . . . .

Section: . . . . .

Number: . . . . .

**Important Instructions**

- Make sure you write your name, number and section number on the exam paper and on the solution booklet.
- Solve all questions (**Q1** through **Q7**). Make sure you show your complete, mathematically correct and neatly written solution.
- You are NOT allowed to share calculators or any other material during the test under any circumstances.
- Cellular phones are NOT allowed to be used in class.

**Q1:** Write the correct definition of each of the following: *(2+2 points)*

- (i) A sequence  $\{q_n\}$  is linearly convergent to  $q$ .
- (ii) A function  $f(x)$  has a zero of multiplicity 5.

**Q2:** Write correct and complete statement for each of the following: *(3+3 points)*

- (i) Fundamental Theorem of Algebra.
- (ii) Fixed Point Theorem.

**Q3:** Let  $f(x) = (x + 2)(x + 1)^2(x - 1)^3(x - 2)$ . To which zero of  $f$  does the Bisection method converge when applied on the interval  $[-3, 0]$ . Justify your answer. *(4 points)*

**Q4:** Let  $f(x) = 1 - 5\frac{\ln(x)}{x}$ . Answer each of the following: *(2+3 points)*

- (i) Show that  $f(x)$  has a zero in the interval  $[1, 20]$ .
- (ii) Use Newton's method with  $p_0 = 20$  and find  $p_1, p_2$ .

**Q5:** Let  $g(x) = \frac{1}{2}(x + \frac{2}{x})$ . Find an interval  $[a, b]$  such that  $g : [a, b] \rightarrow [a, b]$ . Justify your answer. *(4 points)*

**Q6:** Show that  $f(x) = \sin(x) - x$  has a zero of multiplicity three at  $x = 0$ . *(3 points)*

**Q7:** Consider the function  $g(x) = 1 + \sin^2(x)$ , and let  $p_0^{(0)} = 1$ . Use Steffensen's method to find  $p_0^{(1)}$  and  $p_0^{(2)}$ . *(4 points)*

**Total: 30 points**

**Good Luck**