Sultan Qaboos University
Department of Mathematics and Statistics

Math 4141 Fall 2011

Quiz 3 Time: 30 minutes

Name:. . . . . Section: . . . . . Number. . . . . .

In questions 1 and 2, show your complete, mathematically correct and neatly written solution.

Q1: Consider the following integral in answering (i) and (ii) *(4+4 points)*

$$\int_0^2 \frac{dx}{x^4+1}$$

(i) Use the Composite trapezoidal rule with $n = 4$ to approximate the integral.

(ii) If we want the error in the approximation to be less than 10^{-3}, then determine a good choice for h and n.

Hint:

$$\frac{d^2}{dx^2}\left(\frac{1}{x^4+1}\right) = \frac{4x^2(5x^4-3)}{(x^4+1)^3}$$

and the third derivative is zero at

$$x = 0, \quad x = \frac{1}{\sqrt{5}}\left(25 - 10\sqrt{5}\right)^{\frac{1}{4}} \quad \text{and} \quad x = \frac{1}{\sqrt{5}}\left(25 + 10\sqrt{5}\right)^{\frac{1}{4}}.$$

Q2: Answer each of the following: *(3+4 points)*

(i) Suppose that $D = \{(t, y) : \ a \leq t \leq b \ \text{ and } \ -\infty < y < \infty\}$, and consider the initial-value problem

$$y'(t) = f(t, y(t)), \quad a \leq t \leq b, \quad y(a) = \alpha.$$

Under what conditions on $f(t, y)$ we can guarantee that the initial-value problem is well-posed?

(ii) The Legendre polynomial of degree thee is given by $P_3(x) = x\left(x - \sqrt{\frac{3}{5}}\right)\left(x + \sqrt{\frac{3}{5}}\right)$. Therefore, we can write the Guassian quadrature with $n = 3$ as

$$\int_{-1}^{1} f(x)\, dx = c_1 f\left(-\sqrt{\frac{3}{5}}\right) + c_2 f(0) + c_3 f\left(\sqrt{\frac{3}{5}}\right).$$

Find the exact values of c_1, c_2 and c_3. `Hint: You can use MAPLE if you want to.`

Good Luck