Tutorial & Homework Problems

Sultan Qaboos University — College of Science

Department of Mathematics and Statistics

Numerical Analysis — **MATH 4141**

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Homework & Practice Exercise
Chapter 3 +10

Show your complete, mathematically correct, and neatly written solution in each problem you solve.

Suggested Tutorial Problems: In these problems, students are required to solve, or at least, attempt seriously before coming to the tutorial. In the tutorial, we discuss the difficulties you find in these exercises.

Exercises 3.1: Interpolation and the Lagrange Polynomials	1,2,3,8,9,15.
Exercises 3.2: Divided Differences	1,9,12,13,14,15.
Exercises 3.3: Hermite Interpolation	1,2,3,4.
Exercise 10.1: Fixed points for functions of several variables	1,2,3,4.
Exercise 10.2 Newton's Method	1(a) , 1(d).

Computer Assignments: Write a computer code in MAPLE or MATLAB to address each of the following problems:

`Problem 1:` We can use MAPLE to do polynomial interpolations. For example: The command

```
>interp([0,0.6,0.9],[1,cos(0.6),cos(0.9)]);
```

will give you $-.4310868667x^2 - .0324551886x + 1$. This is the interpolating polynomial of degree 2 in question 1:(a), Section 3.1. Now, use MAMPLE to do the assigned exercises in 3.1 and 3.2. Then compare your answers with the answers you obtain using MAPLE.

Graded Homework: Solve the following questions as a graded homework. The due date for submitting your solution is Saturday, November 26, 2011 (class time). Copying someone else's homework is considered cheating/plagiarism and the minimum penalty is zero in the homework. Please read pages 36 and 37 of SQU Undergraduate Academic Regulations (Third Edition, 2005) about the Academic Integrity Policy.

H2Q1: Write TRUE beside the true statement and FALSE beside the false one. You need to give justifications when you write TRUE, and you need to give a counter example when you write FALSE.

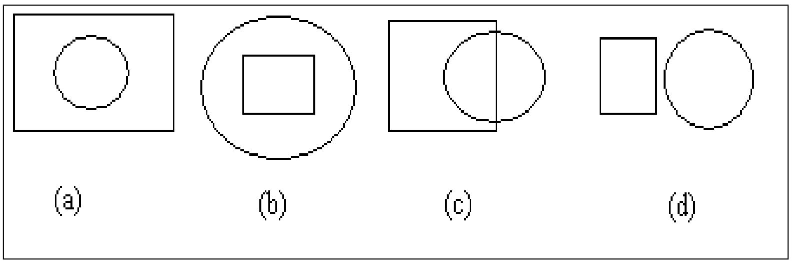
(i) If $f(x)$ is continuous on the interval $[a, b]$ such that $f(a)f(b) > 0$, then $f(x)$ has a solution in the interval $[a, b]$.

(ii) If $f \in C^2([a, b])$ such that $f(c) = 0$ and $f'(c) = 0$ for some $c \in [a, b]$, then $f(x) = (x - c)^2 q(x)$ for some function $q(x)$ that satisfies $q(c) \neq 0$.

(iii) The fixed point iteration $p_{n+1} = g(p_n)$ gives linear convergence all the time.

(iv) If $\{p_n\}_{n=0}^{\infty}$ is a sequence that converges to 0 such that $\lim_{n\to\infty} \left|\frac{p_{n+1}}{p_n}\right| = \lambda$, then p_n converges to 0 of order 1.

H2Q2: You have learned the definitions of linear convergence and super-linear convergence. Write the definition of each of them, then clarify the relationship between the two. In other words, if we give a rectangle for linear convergence and a circle for super-linear convergence, then one of the following diagrams must represent the relationship between the two. Select the correct answer, then either you prove it or you disprove the other ones.



H2Q3: Show that if $p(x)$ is any function that interpolates f at $x_0, x_1, \ldots, x_{n-1}$, and if $q(x)$ is a function that interpolates f at $x_1, x_2, \ldots, x_n$, then the function

$$\frac{(x_n - x)}{(x_n - x_0)} p(x) + \frac{(x - x_0)}{(x_n - x_0)} q(x)$$

interpolates f at the points $x_0, x_1, \ldots, x_n$.

H2Q4: Give a sequence $\{p_n\}$, we define the forward difference for p_n as $\Delta p_n = p_{n+1} - p_n$. Also, we define $\Delta^m p_n$ recursively by

$$\Delta^m p_n = \Delta(\Delta^{m-1} p_n), \quad \text{for } m \geq 2.$$

Now, use this notation to show that

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{1}{h} \Delta f(x_0)$$

$$f[x_0, x_1, x_2] = \frac{1}{2h} = \left(\frac{\Delta f(x_1) - \Delta f(x_0)}{h} \right) = \frac{1}{2h^2} \Delta^2 f(x_0),$$

and in general,

$$f[x_0, x_1, ..., x_k] = \frac{1}{k! h^k} \Delta^k f(x_0).$$

H2Q5: Solve Question 4 in Section 3.3 (page 140).

H2Q6: Solve parts (a) and (b) of Question 5 in Section 10.1 (page 609).

Good Luck