

SULTAN QABOOS UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math3207

Fall 2013

Test 2

Time: 70 minutes

Name: Section: ID Number:

Directions: • This test has 6 compulsory questions. • In each question you solve, you must show your complete, mathematically correct and neatly written solution. • Students are not allowed to share any material during the test. • Cellular phones should not be used in class for any reason.

Group 1: Knowledge Questions

Q1: (2 points each)

- (i) Write down Pascals identity?
- (ii) In how many different ways can n men sit around a circular table?
- (iii) In how many different ways can n objects be placed around a circular table if the orientation is not important?

Group 2: Comprehension & Application Questions

Q2: (2+3 points)

- (i) How many arrangements can be made of the letters in **arrangements**?
- (ii) How many arrangements can be made of the letters in **arrangements** if we want exactly 3 letters between the two letters **r**?

Q3: (3 points)

Find the term that is independent of x in the the expansion of

$$\left(x - \frac{1}{x^2}\right)^{15}.$$

Group 3: Analysis & Application Questions

Q4: What is the coefficient of x^{10} in the expansion of (4 points)

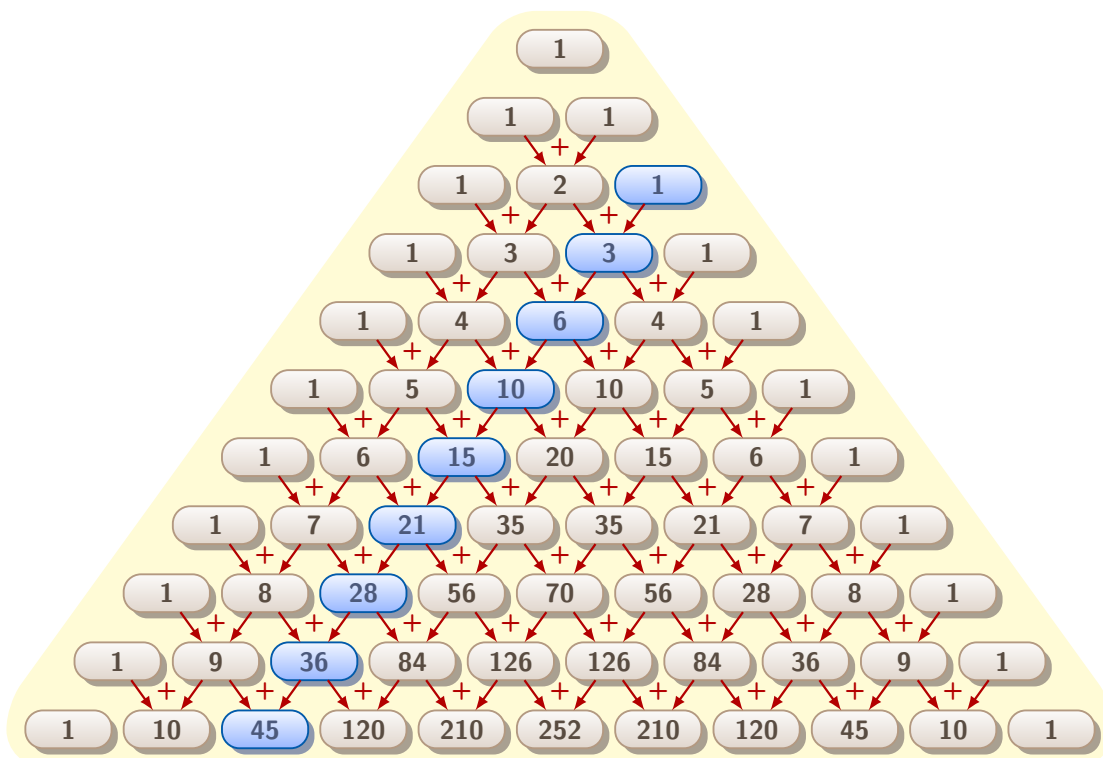
$$(1 - x^2)(1 + 2x)^{20}.$$

Q5: Prove that (4 points)

$$\binom{n}{k} + 2\binom{n}{k-1} + \binom{n}{k-2} = \binom{n+2}{k}.$$

Group 4: Evaluation Questions

Q6: Consider the following highlighted numbers in Pascal's Triangle: (3+5 points)



(i) Consider $P(x) = (1 + x)^n$. Show that the highlighted numbers can be represented by

$$\binom{n}{2}, \quad n = 2, 3, \dots$$

(ii) Find a general formula for the sum of the highlighted numbers.

Hint: Use mathematical induction to prove that

$$\sum_{j=2}^n \binom{j}{2} = \frac{1}{6}n(n^2 - 1), \quad n = 2, 3, \dots$$

End of Exam
Best Wishes