

SULTAN QABOOS UNIVERSITY  
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math3207

Fall 2013

Test 1

Time: 90 minutes

Name: . . . . . Section: . . . . . ID Number: . . . . .

**Directions:** • This test has 10 compulsory questions and 1 optional question (Bonus Question).  
• In each question you solve, you must show your complete, mathematically correct and neatly written solution. • **The bonus question** will be graded as right or wrong without giving partial credits for wrong answers. • Students are not allowed to share any material during the test. • Cellular phones should not be used in class for any reason.

**Group 1: Knowledge Questions**

**Q1:** (3 points)

What does it mean when we say a set of vectors forms basis for  $\mathbf{R}^2$ ?

**Q2:** (2 points)

What does it mean when we say a transformation  $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  is linear?

**Group 2: Comprehension Questions**

**Q3:** (3 points)

Which of the following vectors are along the line  $5x - 3y = 0$ ?

$$V_1 = [3, 5], \quad V_2 = [5, 3], \quad V_3 := [-3, -5], \quad V_4 = [15, 25].$$

**Q4:** (3 points)

Which of the following vectors are perpendicular to the line  $5x - 3y = 0$ ?

$$V_1 = [5, -3], \quad V_2 = [3, 5], \quad V_3 := [-5, 3], \quad V_4 = [50, -30].$$

**Group 3: Application Questions**

**Q5:** (2 points)

Find the projection of  $V_1 = [1, 4]$  in the direction of  $V_2 = [1, 1]$ .

**Q6:** (3 points)

Find the reflection of the point  $(-1, 2)$  about the line  $y = -4x$ .

**Q7:** (2 points)

Find the distance between the point  $(1, 2)$  and the line  $3x + y = 0$ .

Please continue to the next page  $\Rightarrow$

**Group 4: Analysis Questions****Q8:** (3 points)Given a point  $X_0 = (x_0, y_0)$  and a vector  $V = [a, b]$ . Is it true that

$$\begin{cases} x = x_0 + ta \\ y = y_0 + tb \end{cases}$$

gives a parametric representation of the equation of the line that passes through  $X_0$  and parallel to  $V$ ? Justify your answer.**Group 5: Synthesis Questions****Q9:** (4 points)The points  $A = (1, 2), B = (1, 8), C = (7, 2)$  and  $D = (7, 8)$  are the vertices of a polygon.

- (i) Use vectors to show that the polygon is in fact a square.
- (ii) Find a transformation  $T$  that shifts this square and makes it centered at the origin.

**Group 6: Evaluation Questions****Q10:** (5 points)

Show that the shear transformation

$$T([x, y]) = [x + 5y, y + 5x]$$

is a linear transformation, then find the matrix that can represent it.

End of Compulsory Questions

---

**Group 7: Bonus Question****(3 points)**Prove that a linear transformation  $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  maps a line into a line or a point.