

SULTAN QABOOS UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math3207

Fall 2013

Final Exam

Wednesday, January 15, 2014

Time: 150 minutes

Name:

Section:

ID Number:

Directions: • This test has 15 compulsory questions. • In each question you solve, you must show your complete, mathematically correct and neatly written solution. • Students are not allowed to share any material during the test. • Cellular phones should not be used in class for any reason.

Group 1: Knowledge Questions

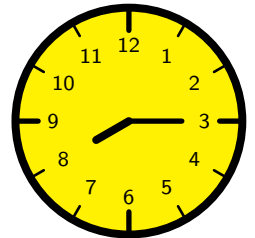
Q1: Give a concise meaning (definition) of each of the following:

- (i) A single variable polynomial of degree n ;
- (ii) A quartic equation in one unknown;
- (iii) Linearly dependent set of vectors;
- (iv) A parallelogram;
- (v) A unit vector.

Q2: Give a complete statement of each of the following theorems.

- (i) Binomial Theorem;
- (ii) Fundamental Theorem of Algebra.

Q1: 10 points
Q2: 5 points



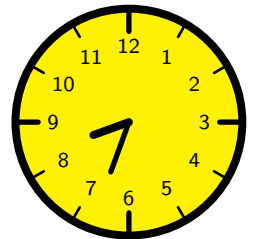
Group 2: Comprehension Questions

Q3: In how many different ways can 10 people be seated around a circular table if two of them (a man and his wife) want to sit together?

Q4: Give an example of a line and a vector so that the vector is perpendicular to the line. The equation of the line must be of the form $y = mx + b$. Also, make sure that you justify your answer.

Q5: Give an example of a cubic equation in the form $x^3 + bx^2 + cx + d = 0$ such that the equation has a zero of multiplicity three at $x = 2$.

Q3: 5 points
Q4: 5 points
Q5: 4 points



Group 3: Application Questions

Q6: Find the projection of the vector $V = [-2, 3]$ in the direction of the vector $U = [1, -2]$.

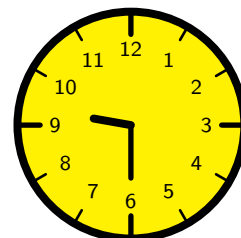
Q7: Find the middle term in the expansion of $(x + \frac{2}{x})^{50}$.

Q8: Find all solutions of the equation $x^6 = 1 + i$.

Q9: Find the base 3 representation of the denary number 3207.

Q10: Without changing into denary form, find the result of $351_6 \times 345_6$.

Q6: 5 points
Q7: 5 points
Q8: 6 points
Q9: 5 points
Q10: 5 points



Group 4: Analysis Questions

Q11: Show that $x = 2$ is a solution of

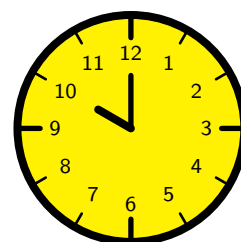
$$x^6 - 9x^5 + 30x^4 - 40x^3 + 48x - 32 = 0$$

with multiplicity 5.

Q12: Let ω^* be a non-real solution of the equation $x^3 + a^3 = 0$, where a is a real number. Show that $\omega^{*2} + a^2 = a\omega^*$.

Q13: Is it true that a linear transformation must take the zero vector into itself? Justify your answer.

Q11: 6 points
Q12: 4 points
Q13: 4 points



Group 5: Evaluation Questions

Q14: Show that a shear transformation $T([x, y]) = [x + ky, y + rx]$ must map a line segment into a line segment, where k and r are constants.

Q15: For all positive integers n , prove that

$$\sum_{k=0}^n \binom{n}{k} 2^{n-k} (-1)^k = 1.$$

Q14: 6 points
Q15: 5 points

