SULTAN QABOOS UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math3110 Fall 2014

Final Exam Time: 150 minutes

Name:. . . . . Section: . . . . . Number. . . . . .

Important Instructions

- Write your name, ID # and Section # on the front cover of your answer booklet.
- You need to show your complete, mathematically correct and neatly written solution.
- You are NOT allowed to share calculators or any other material during the test.
- Cellular phones are NOT allowed to be used for any purpose during the test.
- You should NOT ask the invigilator any questions about the exam.
- You need to solve all questions (**Q1** through **Q10**).

Q1: *(4 points)*
Find all vectors of magnitude $\frac{1}{2}$ that are parallel to $V =< -6, 3, -2 >$.

Q2: *(6 points)*
Find the area of the triangle with vertices $P(1, -1, 0)$, $Q(2, 1, -1)$ and $R(-1, 1, 2)$.

Q3: *(6 points)*
Show that the vector $V =< a, b >$ is orthogonal to the line $ax + by = c$.

Q4: *(8 points)*
Consider the two points $A(-3, 3, -2)$, $B(2, -1, 4)$ and the line L that passes through them.

(i) Find parametric equations that represent L.

(ii) At what point does L intersect the xy-plane?

Q5: *(10 points)*
Find the unit tangent and principal unit normal vectors to $r(t) =< \cos(2t), t, \sin(2t) >$ at the point $(1, 0, 0)$.

Q6: *(10 points)*
Sketch the region defined by the limits of the integration

$$\int_0^2 \int_x^{\sqrt{8-x^2}} (x^2 + y^2)^{\frac{3}{2}} \, dydx,$$

then evaluate the double integral.

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Q7: *(10 points)*

Find the maximum and minimum of the function

$$f(x,y) = x^2y^2 \quad \text{subject to the constraint} \quad x^2 + 4y^2 \leq 24.$$

Q8: *(8 points)*

Given the transformation

$$x = \rho\sin(\phi)\cos(\theta), \quad y = \rho\sin(\phi)\sin(\theta) \quad \text{and} \quad z = \rho\cos(\phi).$$

Find the Jacobian

$$\frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)},$$

and write your answer in simplest form.

Q9: *(8 points)*

Compute the volume of the solid below the surface $z = 1 + e^x \sin y$ and above the region in the xy-plane bounded by the lines $x = -1,\ x = 1,\ y = 0$ and $y = \frac{\pi}{2}$.

Q10: *(10 points)*

Give an accurate statement of Green's theorem. Then use it to evaluate the integral

$$\oint_C (x^3 - y)dx + (x + y^3)dy,$$

where C is the positively oriented curve bounding the region between $y = x^2$ and $y = x$.

`Best Wishes` `Total: 80 points`
