Sultan Qaboos University
Department of Mathematics and Statistics

Math 2108 Spring 2011
Test II Time: 62 minutes

Name:.................... Section: ..... Number........

Important Instructions

- Write your name, ID # and Section # on the front cover of your answer booklet.
- In questions **Q1** to **Q4**, you must show your complete, mathematically correct and neatly written solution. In question **Q5**, circle the choice that best fits the correct answer.
- You are NOT allowed to share calculators or any other material during the test.
- Cellular phones are NOT allowed to be used for any purpose during the test.
- You should NOT ask the invigilator any questions about the exam.

Q1: (a) Evaluate the integral *(7+5 points)*

$$\int \frac{7x^2+20x+4}{3x(x+1)^2}dx.$$

(b) Determine whether the improper integral

$$\int_0^{\frac{\pi}{4}} \frac{3\sec^2(x)}{\sqrt{1-\tan(x)}}dx.$$

converges or diverges. Find the value of the integral if it converges.

Q2: Show that the sequence $a_n = \dfrac{e^n}{1+e^n}$ is increasing and bounded. *(5 points)*

Q3: Test whether each one of the following series is convergent or divergent: *(6+5+5 points)*

$$(a) \sum_{k=1}^{\infty} \frac{2-\tan^{-1}(k)}{k^2+1} \qquad (b) \sum_{k=1}^{\infty} \frac{\sqrt{k^3+2k+3}}{k^3+5k-2} \qquad (c) \sum_{k=2}^{\infty} \frac{(-1)^{k+1}}{k\ln(k)}.$$

Q4: Determine whether the series *(7 points)*

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{3^k}{k\ 4^k}$$

is absolutely convergent, conditionally convergent or divergent.

Q5: **(Multiple Choice Questions)** See page 2. It is attached with your answer booklet.

Name:. Section: . . . . . Number. . . . .

You must keep this page stapled with your answer booklet

Q5: Circle the choice that best fits the correct answer. *(2 points each)*

- One of the following sequences is divergent

  (**A**) $a_n = \frac{1}{n}$ (**B**) $b_n = (-1)^n \frac{2n^3}{n^3+1}$

  (**C**) $c_n = \frac{(-1)^n}{n}$ (**D**) $S_n = \sum_{k=1}^{n} \left(\frac{-1}{5}\right)^k$

- One of the following series is conditionally convergent:

  (**A**) $\sum_{k=1}^{\infty} \frac{1}{k}$ (**B**) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$

  (**C**) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$ (**D**) $\sum_{k=1}^{\infty} (-1)^k$

- One of the following is true:

  (**A**) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{k=1}^{\infty} a_k$ is convergent.
  (**B**) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{k=1}^{\infty} a_k$ is divergent.
  (**C**) If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$,
  (**D**) If $\sum_{k=1}^{\infty} a_k$ is divergent, then $\lim a_k \neq 0$.

- A student needed to test the series $\sum_{k=1}^{\infty} \dfrac{1}{k + |\sin(k)|}$ for convergence/divergence. The student solved the problem as follows:

  `Step 1:` $0 < \dfrac{1}{k + |\sin(k)|} \leq \dfrac{1}{k}$ for all $k \geq 1$.

  `Step 2:` We know that $\sum_{k=1}^{\infty} \dfrac{1}{k}$ is a divergent p-series with $p = 1$.

  `Step 3:` Because $\sum_{k=1}^{\infty} \dfrac{1}{k}$ is divergent, then $\sum_{k=1}^{\infty} \dfrac{1}{k + |\sin(k)|}$ is divergent by the comparison test.

  What do you think about the solution of this student?

  (**A**) The solution is correct (**B**) Step 1 is wrong
  (**C**) Step 2 is wrong (**D**) Step 3 is wrong

- One of the following integrals is improper:

  (**A**) $\displaystyle\int_0^1 \frac{\cos(x)}{1+\sin(x)} dx$ (**B**) $\displaystyle\int_0^1 \frac{\ln(x+1)}{1+\ln(x+1)} dx$

  (**C**) $\displaystyle\int_0^1 \frac{\cos(x)}{1+\cos(\pi x)} dx$ (**D**) $\displaystyle\int_0^1 \frac{\sin(x)}{1+\cos(x)} dx$

Good Luck