Sultan Qaboos University
Department of Mathematics and Statistics

Math2108 Spring 2011

Test I Time: 60 minutes

Name:. . . . . Section: . . . . . Number. . . . . .

Important Instructions

- Write your name, ID # and Section # on the front cover of your answer booklet.
- In questions **Q1** to **Q6**, you must show your complete, mathematically correct and neatly written solution. In question **Q7**, circle the choice that best fits the correct answer.
- You are NOT allowed to share calculators or any other material during the test.
- Cellular phones are NOT allowed to be used for any purpose during the test.
- You should NOT ask the invigilator any questions about the exam.

Q1: Find the area of the region bounded by the curves of $y = \frac{1}{x^2}, x = 4$ and $y = 4$. *(5 points)*

Q2: The region bounded by the curves of $y = \sqrt{25 - x^2}$ and $y = 3$ is revolved about the x-axis. Sketch the region, and find the volume of the generated solid. *(6 points)*

Q3: The region bounded by the curves of $y = (x-2)^2 + 1, x = 1$ and $x = 4$ is rotated about the y-axis. Sketch the region, and find the volume of the generated solid. *(6 points)*

Q4: Find the surface area of the solid generated by revolving the curve of $y = \sqrt{x},\ 2 \le x \le 4$ about the x-axis. *(6 points)*

Q5: Evaluate each of the following integrals. *(6+6 points)*

$$(i)\ \int_0^{\frac{\pi}{4}} \frac{\cos(x)}{\sin^2(x) + 2\sin(x) + 1} dx \qquad (ii)\ \int \frac{\tan^3(x)}{\sec^2(x)}\, dx.$$

Q6: Let n be a positive integer.

(a) Prove that $\int (\ln x)^n\ dx = x\,(\ln x)^n - n \int (\ln x)^{n-1}\ dx$. *(6 points)*

(b) Use the formula in part (a) to evaluate $\int (\ln x)^3\ dx$. *(3 points)*

Q7 Circle the choice that best fits the correct answer. *(6 points)*

(i) The volume of the solid with cross sectional area $A(x) = \pi(x-1)^2,\ 1 \le x \le 7$ is
(**A**) 63π (**B**) $72\pi^2$ (**C**) $\frac{72}{3}\pi$ (**D**) 72π (**E**) None of these

(ii) The arc length of the curve of $y = (x-1)^{\frac{3}{2}}$ from $x = 1$ to $x = 6$ is given by
(**A**) $\int_1^6 \sqrt{1 + (x-1)^3} dx$ (**B**) $\int_1^6 \sqrt{1 + \frac{9}{4}(x-1)^3} dx$ (**C**) $\frac{3}{2}\int_1^6 \sqrt{x - \frac{5}{9}} dx$ (**D**) None of these

(iii) Suppose that f and g are continuous functions that satisfy $f(x) \ge 0$ and $g(x) \le 0$ for all x. The area between the curves of $f(x)$ and $g(x)$ over the interval [a,b] is
(**A**) $\int_a^b [f(x) - g(x)] dx$ (**B**) $\int_a^b [g(x) - f(x)] dx$ (**C**) $\int_a^b [f(x) + g(x)] dx$ (**D**) 0

Good Luck