Sultan Qaboos University
Department of Mathematics and Statistics

Math 2108 — Spring 2011

Final Exam — Time: 2 hours & 30 minutes

Name:.................... Section: ..... ID Number........

Important Instructions

- Write your name, ID # and Section # on the front cover of your answer booklet.
- In questions **Q1** to **Q8**, you must show your complete, mathematically correct and neatly written solution. In question **Q9**, circle the choice that best fits the correct answer.
- You are NOT allowed to share calculators or any other material during the test.
- Cellular phones are NOT allowed to be used for any purpose during the test.
- You should NOT ask the invigilator any questions about the exam.

Total Points: 100

Q1: *(6+6 points)*

(a) Sketch the region enclosed by $y = x^2 + 1, y = x, x = -1$ and $x = 2$, then find its area.

(b) Sketch the region bounded by the curves of $y = 1 + \sqrt{x}, x = 4, x = 9$ and $y = 0$, then find the volume of the solid generated by revolving the region about the y-axis.

Q2: Evaluate each of the following integrals: *(6+6 points)*

$$(a)\ \int \frac{1}{x^2} \sec\left(\frac{1}{x}\right) dx \qquad (b)\ \int \sqrt{9 - x^2}dx$$

Q3: Evaluate each of the following integrals: *(6+6 points)*

$$(a)\ \int_0^{\pi} \sin^4(x)dx \qquad (b)\ \int_1^{\infty} \frac{e^{-x}}{\sqrt{1 - e^{-x}}}dx$$

Q4: Determine whether each of the following series is conditionally convergent, absolutely convergent or divergent. *(5+7 points)*

$$(a)\ \sum_{k=0}^{\infty} \frac{(-1)^{k+1}k!}{3^k} \qquad (b)\ \sum_{k=1}^{\infty} \frac{(-1)^k 2k}{k^2 + 1}.$$

Q5: Consider $f(x) = \ln(x)$ and answer each of the following: *(7+2 points)*

(a) Find the Taylor series of $f(x)$ about $x = 1$, and find the interval of convergence.

(b) Can we use the series obtained in part (a) to represent $\ln(5)$? Justify your answer.

Q6: Use the geometric series representation of $g(x) = \dfrac{1}{1-x}$, $x \in (-1, 1)$ to find the Maclaurin series of *(5+5 points)*

$$(a)\ \frac{1}{1+x^2} \qquad\qquad (b)\ \tan^{-1}(x).$$

Q7: Find the interval and radius of convergence for each of the following: *(6+6 points)*

$$(a)\ \sum_{k=1}^{\infty} \frac{(2x+1)^k}{(k+1)^2} \qquad\qquad (b)\ \sum_{k=1}^{\infty} \left(\frac{4k}{5k+1}\right)^k x^k$$

Q8: *(3+6 points)*

(a) Find a polar equation corresponding to the rectangular equation $x^2 + y^2 = x$.

(b) Sketch the graph of the polar equation $r = 2 + 4\cos(\theta)$.

Q9: Circle the choice that best fits the correct answer. *(2 points each)*

- The arc length of the curve of $x = \sin(y^2)$ from $y = 0$ to $y = \pi$ is given by
  (**A**) $\int_0^\pi \sqrt{1 + 4x^2\cos(x)}dx$ (**B**) $\int_0^\pi 2\pi y\sqrt{1 + 4y^2\sin(y)}dy$

  (**C**) $\int_0^\pi \sqrt{1 + 4y^2\cos^2(y^2)}dy$ (**D**) None of the above.

- The series $\sum_{k=0}^{\infty} \frac{3^k + 4(-2)^k}{6^k}$ is

  (**A**) convergent to zero (**B**) divergent
  (**C**) convergent to 5 (**D**) none of the above.

- One of the following is a non-power series
  (**A**) $\sum_{k=1}^{\infty} x^k$ (**B**) $\sum_{k=1}^{\infty} k^{\frac{1}{k}}(x-1)^k$

  (**C**) $\sum_{k=1}^{\infty} 3k!\cos(x^k)$ (**D**) $\sum_{k=1}^{\infty} \sin(k)(x+5)^k$

- The polar point $(r, \theta) = (1, \frac{\pi}{2})$ has the following rectangular representation
  (**A**) $(1, \frac{5\pi}{2})$ (**B**) $(0, 1)$ (**C**) $(1, \frac{-3\pi}{2})$ (**D**) none of these.

- The rectangular point $(x, y) = (1, -1)$ has the following polar representation
  (**A**) $(-\sqrt{2}, \frac{-\pi}{4})$ (**B**) $(\sqrt{2}, \frac{\pi}{4})$ (**C**) $(\sqrt{2}, \frac{-\pi}{4})$ (**D**) none of these.

- One of the following sequences is convergent
  (**A**) $a_n = (-1)^n$ (**B**) $b_n = \sin(n)$ (**C**) $c_n = \sqrt{n} - \sqrt{n+1}$ (**D**) $s_n = \cos(n)$

Good Luck