SULTAN QABOOS UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS
May 16 2008

BUSINESS MATHEMATICS I (MATH1101)
Spring 2009, Final Exam
(Time allowed: 150 minutes)

NAME:________________________________ **ID#:**________ **Section:**________

INSTRUCTIONS: Please read these instructions before you start solving.

- Write your name, ID number and Section number in the first page and ID number at the top of each sheet.
- You need to show your complete, mathematically correct and neatly written work.
- It is prohibited to exchange calculators or share any material during the exam.
- You may use the back side of the page if needed.
- Please keep the sheets stapled.

Question	points	score
Q1	17 pts	
Q2	12 pts	
Q3	16 pts	
Q4	12 pts	
Q5	11 pts	
Q6	10 pts	
Q7	8 pts	
Q8	14 pts	
TOTAL	100 pts	

Solve each of the following questions. In questions **Q1** to **Q7**, you need to show your complete, mathematically correct and neatly written work.

Q1: *(5+4+3+5=17 points)*

(a) Solve each one of the following equations for x.

(i) $\left|\dfrac{-3x-2}{x+2}\right| = 5$ $\quad (ii)$ $(3)^{x^2} = 9^{1-x}$ $\quad (iii)$ $\begin{vmatrix} 1 & 0 & 0 \\ x^2 & x+3 & 2 \\ x & x & x+1 \end{vmatrix} = 3$

`Solution:`

(i)

$$\begin{aligned} \frac{-3x-2}{x+2} = 5 \quad &\text{or} \quad \frac{-3x-2}{x+2} = -5 \\ -3x-2 = 5x+10 \quad &\text{or} \quad -3x-2 = -5x-10 \\ -12 = 8x \quad &\text{or} \quad 8 = -2x \\ x = \frac{-3}{2} \quad &\text{or} \quad x = -4 \end{aligned}$$

(ii)

$$\begin{aligned} 3^{x^2} &= (3^2)^{1-x} \\ x^2 &= 2-2x \\ x^2+2x-2 &= 0 \\ x &= \frac{-2 \pm \sqrt{4-4(-2)}}{2} \\ x &= -1 \pm \sqrt{3} \end{aligned}$$

(iii)

$$\begin{aligned} (x+3)(x+1) - 2x &= 3 \\ x^2+3x+x+3-2x &= 3 \\ x^2+2x &= 0 \\ x(x+2) &= 0 \\ x = 0 \quad &\text{or} \quad x = -2 \end{aligned}$$

(b) Solve the inequality $3 + 2|3-2x| < 7$ and write your solution set in interval form.

`Solution:`

$$\begin{aligned} 2|3-2x| &< 4 \\ |3-2x| &< 2 \\ -2 < 3-2x &< 2 \\ -5 < -2x &< -1 \end{aligned}$$

Hence, $\frac{5}{2} > x > \frac{1}{2}$ and the solution set is $(\frac{1}{2}, \frac{5}{2})$.

Q2: *(4+4+4=12 points)*

(a) Find the domain of $f(x) = \frac{1}{\sqrt{x^2-5x-6}}$.

`Solution:` We need $x^2 - 5x - 6 > 0$ or $(x-6)(x+1) > 0$.
$x^2 - 5x - 6 > 0$ when $x < -1$ or $x > 6$.
Hence, the domain is $(-\infty, -1) \cup (6, \infty)$.

(b) Find the domain and range of $g(x) = 2 + e^x$.

`Solution:` Domain: All real numbers. Range: $(2, \infty)$.

(c) Find the inverse of $g(x) = 2 + e^x$.

`Solution:`

$$y = 2 + e^x \quad \Rightarrow \quad e^x = y - 2 \quad \Rightarrow \quad x = \ln(y-2).$$

Hence, $g^{-1}(x) = \ln(x-2)$.

Q3: *(3+4+5+4=16 points)*

(a) Find the equation of the circle whose radius is 3 that lies in the first quadrant and touches both axes.

`Solution:` The center of the circle is $(3, 3)$. So, the equation is

$$(x-3)^2 + (y-3)^2 = 3^2.$$

(b) Determine the maximum or minimum value of the function $f(x) = 2x^2 + x - 5$.

`Solution:` This parabola opens upward. The minimum is

$$f(\frac{-b}{2a}) = f(\frac{-1}{4}) = \frac{2}{16} - \frac{1}{4} - 5 = -\frac{41}{8}.$$

(c) Find the equation of the line passing through $(1, 2)$ and is parallel to $4x - 2y = -3$.

`Solution:` Since the line is parallel to $4x - 2y + 3 = 0$, then it has a slope $\frac{4}{2} = 2$.
Now, the equation is

$$y - 2 = 2(x-1) \quad \Rightarrow \quad y = 2x.$$

(d) Find the x and y intercepts of the line $4x - 2y + 3 = 0$.

`Solution:` x-intercept $(\frac{-3}{4}, 0)$ y-intercept $(0, \frac{3}{2})$.

Q4: Given the system of linear equations *(2+1+5+4=12 points)*

$$\begin{aligned} x - 2y &= 2 \\ -x + 2z &= 2 \\ 2y - z &= 0 \end{aligned}$$

(a) Write the above system in the form $AX = B$.

Solution:

$$\begin{bmatrix} 1 & -2 & 0 \\ -1 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

(b) Write the augmented matrix of the system.

Solution:

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 2 \\ -1 & 0 & 2 & 2 \\ 0 & 2 & -1 & 0 \end{array}\right]$$

(c) Use the method of row reduction to solve the system.

Solution:

$$R_2 + R_1 \Rightarrow R_2 \left[\begin{array}{ccc|c} 1 & -2 & 0 & 2 \\ 0 & -2 & 2 & 4 \\ 0 & 2 & -1 & 0 \end{array}\right]$$

$$\frac{-1}{2} R_2 \Rightarrow R_2 \left[\begin{array}{ccc|c} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 2 & -1 & 0 \end{array}\right]$$

$$-2R_2 + R_3 \Rightarrow R_3 \left[\begin{array}{ccc|c} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$R_3 + R_2 \Rightarrow R_2 \left[\begin{array}{ccc|c} 1 & -2 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$2R_2 + R_1 \Rightarrow R_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Hence, $z = 4, y = 2$ and $x = 6$.

(d) Use Cramer's rule to find the x value.

Solution:

$$x = \frac{\begin{vmatrix} 2 & -2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 0 \\ -1 & 0 & 2 \\ 0 & 2 & -1 \end{vmatrix}} = \frac{2(-4) + 2(-2) + 0}{1(-4) + 2(1) + 0} = \frac{-8 - 4}{-4 + 2} = 6.$$

CONTINUED

Q5: Consider the matrix *(5+3+3=11 points)*

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 1 & 4 & 3 \\ -5 & 6 & 0 \end{bmatrix}$$

(a) Find the adjoint of the matrix A.

`Solution:`

$$Adj(A) = \begin{bmatrix} \begin{vmatrix} 4 & 3 \\ 6 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ -5 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ -5 & 6 \end{vmatrix} \\ -\begin{vmatrix} -3 & 0 \\ 6 & 0 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 5 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & -3 \\ -5 & 6 \end{vmatrix} \\ \begin{vmatrix} -3 & 0 \\ 4 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} -18 & -15 & 26 \\ 0 & 0 & 3 \\ -9 & -6 & 11 \end{bmatrix}^T$$

$$= \begin{bmatrix} -18 & 0 & -9 \\ -15 & 0 & -6 \\ 26 & 3 & 11 \end{bmatrix}$$

(b) Evaluate the determinant of A.

`Solution:`

$$det(A) = 2\begin{vmatrix} 4 & 3 \\ 6 & 0 \end{vmatrix} + 3\begin{vmatrix} 1 & 3 \\ -5 & 0 \end{vmatrix} + 0 = -36 + 45 = 9.$$

(c) Is the matrix A invertible? If yes, find A^{-1}.

`Solution:` Since $det(A) \neq 0$, then the matrix is invertible and

$$A^{-1} = \frac{Adj(A)}{det(A)} = \frac{1}{9}\begin{bmatrix} -18 & 0 & -9 \\ -15 & 0 & -6 \\ 26 & 3 & 11 \end{bmatrix}$$

Q6: *(4+6=10 points)*

(a) Suppose that 1000 R.O. is invested at a 6% compounded semi-annually, how long does it take the investment to increase to 3000 R.O.

`Solution:` We need to find n such that

$$1000(1+\frac{6\%}{2})^{2n} = 3000$$

Now, we solve for n

$$\begin{aligned} (1.03)^{2n} &= 3 \\ 2n &= \frac{\ln 3}{\ln 1.03} \\ n &= \frac{1}{2}\frac{\ln 3}{ln 1.03} \approx 18.583. \end{aligned}$$

So, it takes about 19 years to obtain 3000 R.O.

(b) A dealer can sell 200 units of certain commodity per day at 30 R.O. and 250 units at 27 R.O. per day. The supply equation for that commodity is $6p = x+48$. Find the demand equation assuming it to be linear, then find the equilibrium price and quantity.

`Solution:` The demand equation has to pass through the points $(200, 30)$ and $(250, 27)$. So, the slope of the line is $\frac{27-30}{250-200} = \frac{-3}{50}$.
The demand equation is

$$\begin{aligned} p - 30 &= \frac{-3}{50}(x - 200) \\ p &= \frac{-3}{50}x + 42. \end{aligned}$$

To find the equilibrium point, we solve the two equations $p = \frac{-3}{50}x + 42$ and $p = \frac{1}{6}x + 8$. By substitution, we obtain

$$\begin{aligned} \frac{-3}{50}x + 42 &= \frac{1}{6}x + 8 \\ \frac{-68}{6(50)}x &= 8 - 42 \\ &= -34. \end{aligned}$$

Hence, $x = 150$ units is the equilibrium quantity. We substitute in one of the equations to obtain the equilibrium price. $p = \frac{150}{6} + 8 = 33$ R.O.

Q7: *(4+4=8 points)*

A manufacturer can sell x units of a product each week at a price of p Rials per unit, where $p = 200 - x$. It costs $(2800 + 45x)$ Rials to produce x units.

(a) How many units should be sold each week to generate a revenue of 9600 Rials?

`Solution:` Since the revenue equals $x(200 - x)$, then

$$\begin{aligned} 9600 &= (200 - x)x \\ x^2 - 200x + 9600 &= 0 \\ (x - 80)(x - 120) &= 0 \end{aligned}$$

Hence, $x = 80$ units or $x = 120$ units.

(b) How many units should the manufacturer produce and sell each week to obtain a profit of 3200 Rials?

`Solution:` Since Profit = Revenue - Cost, then we need to solve

$$3200 = xp - (2800 + 45x) = x(200 - x) - 2800 - 45x.$$

We obtain $x^2 - 155x + 6000 = 0$ or $(x - 75)(x - 80) = 0$.
Hence $x = 75$ or 80 units.

Q8: Circle the correct choice (You need to circle one choice only, and you are not required to show your complete work). *(2 points each)*

(i) The slope of the line $4x - 2y + 7 = 0$ is

(a) 2 **(b)** -2 **(c)** 4 **(d)** $\frac{1}{2}$ **(e)** $\frac{-1}{2}$

(ii) If $f(x)$ has an inverse, then the graph of $f^{-1}(x)$ is the reflection of the graph of $f(x)$ about

(a) x-axis
(b) y-axis
(c) y=x
(d) None of the above.

(iii) $x^2 + y^2 - 4x + 6y - 12 = 0$ is a circle whose center and radius are

(a) $(-2, -3),\ r = 25$
(b) $(-2, -3),\ r = 5$
(c) $(2, -3),\ r = 25$
(d) $(2, -3),\ r = 5$
(e) $(-2, 3),\ r = 5$

(iv) $\log x^3 - \log y^2$ equals

(a) $\frac{3 \log x}{2 \log y}$
(b) $\frac{\log x^3}{\log y^2}$
(c) $\log(x^3 - y^2)$
(d) $\log \frac{x^3}{y^2}$
(e) none of the above.

(v) If $f(x) = \frac{1}{2-x}$ and $g(x) = \sqrt{x^2 - 1}$, then $f \circ g(\sqrt{2}) =$

(a) undefined.
(b) -1
(c) $\frac{1}{2-\sqrt{3}}$
(d) 4
(e) None of the above.

(vi) The graph of the function $f(x) = 3 - 2x^2 + 4x$ is called

(a) a line
(b) a circle
(c) a parabola
(d) an ellipse

(vii) The sum of the first 20 terms of $2 + 7 + 12 + 17 + 22 + \cdots$ is

(a) ar^{20}
(b) 97
(c) 102
(d) 990
(e) none of the given choices.

End of Questions
Good Luck