SULTAN QABOOS UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS
May 2 2008

BUSINESS MATHEMATICS I (MATH1101)

Spring 2009, Second Exam

(Time allowed: 60 minutes)

NAME:____________________________________ **ID#:**_________ **Section:**________

INSTRUCTIONS: Please read these instructions before you start solving.

- Write your name, ID number and Section number in the first page and ID number at the top of each sheet.
- You need to show your complete, mathematically correct and neatly written work.
- It is prohibited to exchange calculators or share any material during the exam.
- You may use the back side of the page if needed.
- Please keep the sheets stapled.

Question	points	score
Q1	4 pts	
Q2	6 pts	
Q3	7 pts	
Q4	9 pts	
Q5	8 pts	
Q6	6 pts	
TOTAL	40 pts	

Solve each of the following questions. You need to show your complete, mathematically correct and neatly written work.

Q1: **i:** Determine the values of u, v, and w so that *(3 points)*

$$\begin{bmatrix} 4 & u & 3 \\ v & -1 & 2 \end{bmatrix} = \begin{bmatrix} v-1 & 2-u & 3 \\ 5 & w+1 & 2 \end{bmatrix}.$$

`Solution:` From the entries a_{11} and a_{21}, we obtain $v = 5$. From the entry a_{12}, we obtain $2 - u = u$ or $u = 1$. From the entry a_{22}, we obtain $w + 1 = -1$ or $w = -2$. Hence,

$$v = 5, \quad u = 1, \quad w = -2.$$

ii: Find the size or dimension of the matrices in part **i**. *(1 point)*

`Solution:` Both matrices are of size 2×3.

Q2: Given *(6 points)*

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \quad \textit{and} \quad B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}.$$

Is it true that $A^2 - B^2 = (A - B)(A + B)$? Justify your answer.

`Solution:` Since $(A-B)(A+B) = A^2+AB-BA-B^2$, then $A^2-B^2 = (A-B)(A+B)$ if and only if $AB - BA = 0$ if and only if $AB = BA$.

Now, since

$$AB = \begin{bmatrix} 8 & -3 \\ 5 & -2 \end{bmatrix}, \qquad \text{and} \qquad BA = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix},$$

then $AB \neq BA$, and hence $A^2 - B^2 \neq (A - B)(A + B)$.

`Remark:` Students may find $(A - B)$ and $(A + B)$, then multiply to compare with $A^2 - B^2$. In this case, use your discretion to distribute the 6 points.

Q3: **i:** Find the domain of $f(x) = \frac{\ln x}{1-\ln x}$. *(2 points)*

`Solution:` Since we need $\ln x \neq 1$ and $x > 0$, then the domain is

$$\{x : \ x > 0 \text{ and } x \neq e\} = (0, e) \cup (e, \infty).$$

ii: Solve $\log_x(1 - x) - \log_x 6 = 2$. *(5 points)*

`Solution:`

$$\begin{aligned} \log_x \frac{(1-x)}{6} = 2 \quad &\Rightarrow \quad \frac{1-x}{6} = x^2. \\ &\Rightarrow \quad 6x^2 + x - 1 = 0 \\ &\Rightarrow \quad (2x+1)(3x-1) = 0 \\ &\Rightarrow \quad x = \frac{-1}{2} \quad \text{or} \quad x = \frac{1}{3}. \end{aligned}$$

Since the base of a logarithmic function is positive, we take $x = \frac{1}{3}$. and neglect the negative value.

CONTINUED

Q4: **i:** Find the sum of *(4 points)*

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

`Solution:` We have the infinite sum of a geometric progression with $a = 1$ and $r = \frac{1}{2}$. Thus,

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = (1)\frac{(\frac{1}{2})^{n+1} - 1}{\frac{1}{2} - 1},$$

where n is very large ($n \to \infty$). Hence, the sum is

$$\frac{(\frac{1}{2})^{n+1} - 1}{\frac{1}{2} - 1} = \frac{2(1-0)}{1} = 2.$$

ii: Find the tenth term of the geometric progression whose second and fifth terms are 24 and 81 respectively. *(5 points)*

`Solution:` A geometric progression is of the form

$$a, ar, ar^2, \ldots, ar^{n-1}.$$

The second term is $ar = 24$ and the fifth term is $ar^4 = 81$. Thus, we need to solve the equations

$$ar = 24 \quad \text{and} \quad ar^4 = 81.$$

$$\begin{aligned} (ar)r^3 &= 81 \\ 24r^3 &= 81 \\ r^3 &= \frac{81}{24} = \frac{27}{8} \quad \Rightarrow \quad r = \frac{3}{2} \end{aligned}$$

From the equation $ar = 24$, we obtain $a\frac{3}{2} = 24$, and consequently $a = 16$.
Hence, the tenth term is $ar^9 = 16(\frac{3}{2})^9 = \frac{3^9}{2^5} = \frac{19683}{32}$.

Q5: *(4 points each)*

i: Which is better for the investor a semi-annual compounding with nominal rate 8%, or an annual compounding at 8.1%. Justify your answer.

`Solution:` We find and compare the effective rate of both investments.
For the semi-annual one $i_{eff} = (1 + \frac{0.08}{2})^2 - 1 = 0.0816$.
For the annual one $i_{eff} = (1 + 0.08) - 1 = 0.081$.
Hence, a semi-annual compounding with nominal rate 8% is better for the investor than 8.1% compounded annually.

`Remark:` students may compare $p(1 + \frac{8\%}{2})^{2n}$ with $p(1 + 8.1\%)^n$. In this case, use your discretion to distribute the 4 points.

ii: At what nominal rate of interest does money double in value in 10 years if compounded continuously?

`Solution:` Let the nominal rate of interest be i. If the money double, then

$$\begin{aligned} pe^{10i} &= 2p \\ e^{10i} &= 2 \\ \ln(e^{10i}) &= \ln 2 \\ 10i &= \ln 2 \\ i &= \frac{\ln 2}{10} = 6.93\%. \end{aligned}$$

Q6: *(6 points)*

Mohammed borrowed 1800 Rials from his brother, and promised to pay his brother back in monthly installments. Each installment is more than the previous one by 10 Rials. If the amount of the first instalment is 50 Rials, then how long does it take Mohammed to pay the 1800 Rials back?

`Solution:`

At the end of the first month, Mohammed pays 50 Rials back.
At the end of the second month, Mohammed pays 50+10 Rials back.
At the end of the third month, Mohammed pays 50+2(10) Rials back.
. . . . .
At the end of the nth month, Mohammed pays $50 + 10(n - 1)$ Rials back.

Thus, we need to find n such that

$$50 + (50 + 10) + (50 + 2(10)) + (50 + 3(10)) + \ldots + (50 + 10(n - 1)) = 1800.$$

$$\begin{aligned} 50n + 10(1 + 2 + 3 + \ldots + n - 1) &= 1800 \\ 50n + 10\frac{(n-1)n}{2} &= 1800 \\ 5n^2 - 5n + 50n - 1800 &= 0 \\ n^2 + 9n - 360 &= 0 \\ (n - 15)(n + 24) &= 0 \end{aligned}$$

Hence, $n = 15$ or $n = -24$, but n must be positive. So, it takes Mohammed 15 months to pay the loan back to his brother.

End of Questions
Good Luck