SULTAN QABOOS UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS
March 24 2008

BUSINESS MATHEMATICS I (MATH1101)

Spring 2009, First Exam

(Time allowed: 60 minutes)

NAME: Solution   ID#:_____ Section:_____

INSTRUCTIONS: Please read these instructions before you start solving.

- Write your name, ID number and Section number in the first page and ID number at the top of each sheet.
- For part A of this exam, you need to show your complete, mathematically correct and neatly written work.
- For part B, you need to circle one choice only, and you are not required to show your complete work.
- It is prohibited to exchange calculators or share any material during the exam.
- You may use the back side of the page if needed.
- Please keep the sheets stapled.

Question	points	score
Q1	3 pts	3
Q2	3 pts	3
Q3	6 pts	6
Q4	4 pts	4
Q5	9 pts	9
Q6-Q10	15 pts	15
TOTAL	40 pts	40

Part A

Solve each of the following questions. You need to show your complete, mathematically correct and neatly written work.

Q1: Find the domain of the function *(3 points)*

$$f(x) = \frac{\sqrt{2-x}}{|x-3|}.$$

Solution: We need $2 - x \geq 0$ and $x \neq 3$, so the domain is

$$(-\infty, 2].$$

Q2: Given the function *(3 points)*

$$g(x) = \begin{cases} \frac{1}{x-2} & x < 2 \\ 3x + 5 & x \geq 2. \end{cases}$$

Evaluate each of $g(1), g(2), g(3)$.
Solution:

$$g(1) = \frac{1}{1-2} = -1.$$
$$g(2) = 3(2) + 5 = 11.$$
$$g(3) = 3(3) + 5 = 14.$$

Q3: *(6 points)*

Solve the inequality $6x - 20 \leq 3x + 1 < 5x - 7$, and write the solution in interval notation.
Solution: We need

$$6x - 20 \leq 3x + 1 \qquad \text{AND} \qquad 3x + 1 < 5x - 7$$

$$3x \leq 21 \qquad \text{AND} \qquad 8 < 2x$$

$$x \leq 7 \qquad \text{AND} \qquad x > 4.$$

Hence, the solution set is $(4, 7]$.

CONTINUED

Q4: *(4 points each)*

The cost of producing x units of a certain commodity per day is given by $y_c = 1000 + 5x$. If each item can be sold for 10 R.O., then determine the break-even point.

Solution: Break-even means that Revenue = Cost, so

$$\begin{aligned} 10x &= 1000 + 5x \\ 5x &= 1000 \\ x &= 200 \text{ units.} \end{aligned}$$

The break-even point is $(200, 2000)$.

Q5: *(9 points)*

The profit $p(x)$ in rials obtained by manufacturing and selling x units of certain product is given by $p(x) = 60x - x^2$.

(a) What is the profit of manufacturing and selling 10 units?

Solution: $p(10) = 60(10) - 10^2 = 600 - 100 = 500$ Rials.

(b) Determine the number of units that must be produced and sold to maximize the profit.

Solution: The profit is maximized at

$$x = \frac{-b}{2a} = \frac{-60}{-2} = 30 \text{ items.}$$

(c) What is the maximum profit?

Solution: The maximum profit is

$$p(30) = 60(30) - 30^2 = 1800 - 900 = 900 \text{ Rials.}$$

(d) How many units must be manufactured and sold to obtain a profit of at least 800 Rials?

Solution: To obtain a profit of at least 800 Rials means $p(x) \geq 800$. So we need to solve the inequality

$$60x - x^2 \geq 800,$$

or equivalently

$$x^2 - 60x + 800 \leq 0.$$

We solve the equation

$$\begin{aligned} x^2 - 60x + 800 &= 0 \\ (x - 20)(x - 40) &= 0. \end{aligned}$$

So, $x = 20$ or $x = 40$. Now, we use three test points to test the signs

x	0	30	100
$x^2 - 60x + 800$	800	-100	4800

$\underline{+ + + + + + + \; 20 \quad \text{- - - - - - - - - - -} \quad 40 \quad + + + + + + + + +}$

The number of items that must be manufactured and sold to obtain a profit of at least 800 Rials must be between 20 and 40 items inclusive, i.e. $[20, 40]$.

Part B

Circle the correct choice. *(3 points each)*

Q6: The domain of a polynomial function is the set of
   (a) integer numbers   (b) fraction numbers $\boxed{\text{(c) real numbers}}$
   (d) natural numbers   (e) positive numbers

Q7: We can define a function as a relationship between two sets, one of them is called domain and the other is called range such that
   (a) each element in the domain is related to at least one element in the range.
   (b) each element in the range is related to at least one element in the domain.
   (c) $\boxed{\text{each element in the domain is related to exactly one element in the range.}}$
   (d) each element in the domain is related to at most one element in the range.
   (e) each element in the range is related to at most one element in the domain.

Q8: The solution of the inequality $|x| \geq 5$ is
   (a) $[5, \infty)$ (b) $[-5, \infty)$ (c) $[-5, 5]$
   (d) $\boxed{[5, \infty) \cup (-\infty, -5]}$ (e) none of the above.

Q9: The equation $x^2 + y^2 - 4x + 6y - 12 = 0$ is an equation of
   (a) a line   (b) a parabola
   (c) $\boxed{\text{a circle with radius 5 and center } (2, -3)}$
   (d) a circle with radius 25 and center $(2, -3)$
   (e) a circle with radius 5 and center $(-2, 3)$

Q10: The system of equations

$$3x - 2y = 1$$
$$-6x + 4y = 3$$

has
   (a) exactly one solution.   (b) two solutions.
   (c) infinite number of solutions.   (d) $\boxed{\text{no solution.}}$
   (e) too many solutions.

End of Questions
Good Luck