> restart :

We define C1,C2,C3,C4,C5,C6 and C7 that we have obtained in the Jordan Normal Form before equation (5.4).

> $C1 := \frac{(1-p)}{2\cdot q}\cdot\left(\frac{2}{r}\cdot(1-p)\cdot(y+1)+\mathrm{mu}-1\right):$

$C2 := \frac{q}{r}\cdot(1+y):$

$C3 := \frac{2}{r}\cdot(1-p)\cdot(y+1)+\frac{1}{2}\cdot(\mathrm{mu}-1):$

$C4 := \frac{(1-p)^3}{q\cdot r}:$

$C5 := \frac{(1-p^2)}{r}:$

$C6 := \frac{3\cdot q}{r}\cdot(1-p):$

$C7 := \frac{3}{r}\cdot(1-p)^2:$

Now, we define the expresssions of C20, C11, C02 and C21 in equation (5.4)

> $C20 := \frac{1}{8}\cdot\left(2\cdot C3-\mathrm{mu}-1+2\cdot\left(C1-C2+\frac{q}{y}\right)\cdot I\right):$

$C11 := \frac{1}{4}\cdot(-1-\mathrm{mu}+2\cdot(C1+C2)\cdot I):$

$C02 := \frac{1}{8}\cdot\left(-2\cdot C3-\mathrm{mu}-1+2\cdot\left(C1-C2-\frac{q}{y}\right)\cdot I\right):$

$C21 := \frac{1}{8}\cdot(C7+3\cdot C5+(3\cdot C4+C6)\cdot I):$

We evaluate each term in the expression of A in equation (5.4)

> $R1 := evalc\left(\mathrm{Re}\left(\left(2\cdot p^2-p-\frac{3}{2}+\frac{q\cdot(1-6\cdot p+4\cdot p^2)}{2\cdot(1-p)}\cdot I\right)\cdot C11\cdot C20\right)\right):$

$R2 := \frac{1}{2}\cdot evalc(\mathrm{abs}(C11))^2:$

$R3 := evalc((\mathrm{abs}(C02))^2):$

$R4 := evalc(-\mathrm{Re}((p-q\cdot I)\cdot C21)):$

We add the terms of A* in *equation* (5.4), then *simplify the expression* and *multiply it* by *the positive factor* 64$\cdot\left((1-p)\,q^2r^2y^2\right)$ *so we can get rid of the denominator.

> $G1 := (p, q, y, \mathrm{mu}, r) \rightarrow \left(64\cdot\left((1-p)\,q^2r^2y^2\right)\right)\cdot simplify(R1+R2+R3+R4):$

> collect(G1(p, q, y, mu, r), q, factor);

$$\left(-8\mu p^2 r y^3+16 p^3 y^4+8\mu p^2 r^2 y-8\mu p^2 r y^2+12\mu p r y^3+16 p^3 r y^2+32 p^3 y^3+8 p^2 r y^3-56 p^2 y^4-12\mu p r^2 y+12\mu p r y^2-2\mu r y^3+16 p^3 r y+16 p^3 y^2+8 p^2 r^2 y-40 p^2 r y^2-112 p^2 y^3-12 p r y^3+48 p y^4+2\mu r^2 y-2\mu r y^2-24 p^2 r y-56 p^2 y^2-12 p r^2 y+24 p r y^2+96 p y^3+2 r y^3-8 y^4-4 p r^2-12 p r y+48 p y^2+2 r^2 y-2 r y^2-16 y^3+4 r^2+20 r y-8 y^2\right) q^4-y(-1+p)\left(4\mu^2 p^2 r^2 y+24\mu p^3 r y^2\right. \quad (1)$$

$$-32p^4y^3-6\mu^2pr^2y+8\mu p^3r^2+24\mu p^3ry+16\mu p^2r^2y-44\mu p^2ry^2-16p^4ry$$
$$-64p^4y^2-24p^3ry^2+112p^3y^3+7\mu^2r^2y-12\mu p^2r^2-44\mu p^2ry-16\mu pr^2y$$
$$-10\mu pry^2-16p^4r-32p^4y-8p^3r^2+40p^3ry+224p^3y^2-4p^2r^2y+76p^2ry^2$$
$$-112p^2y^3+2\mu pr^2-10\mu pry+30\mu ry^2+40p^3r+112p^3y+12p^2r^2+24p^2ry$$
$$-224p^2y^2+14pr^2y-70pry^2+16py^3+2\mu r^2+30\mu ry-28p^2r-112p^2y$$
$$-2pr^2-46pry+32py^2-7r^2y+18ry^2+16y^3+16py-2r^2-2ry+32y^2+4r$$
$$+16y\Big)q^2+2y^2(p+1)(2p-3)(-1+p)^3(\mu r-2py-2p-r+2y+2)^2$$

> $G2 := (p,q,y,\mathrm{mu},r) \to \Big(-24p^2yr+16p^3yr+16p^3y^2-4pr^2+2yr^2-2y^3\mu r$
$-12yp\mu r^2+8p^2\mu r^2y-12ypr^2+8p^2r^2y+12y^3p\mu r+12y^2p\mu r-8p^2y^3\mu r$
$-8p^2y^2\mu r+96py^3+48y^4p+2y^3r-56y^4p^2-112y^3p^2-2y^2r+16p^3y^4$
$+32p^3y^3+2r^2\mu y-16y^3-12y^3pr-2y^2\mu r+24y^2pr+8p^2y^3r-40p^2y^2r$
$+16p^3y^2r-8y^4-12ypr-8y^2+48py^2+20yr-56p^2y^2+4r^2\Big)q^4-y(-1$
$+p)\Big(4r+24p^2yr+2\mu r^2p+40p^3yr-12p^2\mu r^2+224p^3y^2+112p^3y-28p^2r$
$+2\mu r^2-2pr^2-64p^4y^2-32p^4y+40p^3r+12p^2r^2-7yr^2-44p^2y\mu r+24p^3y\mu r$
$-10yp\mu r-16p^4r-16yp\mu r^2+4p^2\mu^2r^2y+16p^2\mu r^2y-6p\mu^2r^2y+14ypr^2$
$-4p^2r^2y+7\mu^2r^2y-8p^3r^2-10y^2p\mu r-44p^2y^2\mu r+24p^3y^2\mu r+16py^3$
$-112y^3p^2+18y^2r+112p^3y^3-32p^4y^3+16y^3+30y^2\mu r-70y^2pr+76p^2y^2r$
$-24p^3y^2r+8p^3\mu r^2-16p^4yr+30y\mu r-46ypr+16py-112p^2y+32y^2$
$+32py^2-2yr-224p^2y^2-2r^2+16y\Big)q^2+2y^2(p+1)(2p-3)(-1$
$+p)^3(2y+2-2py-2p+\mu r-r)^2:$

Next, we manipulate the expression that we denote G2 by following the ideas given in the paragraph followed equation (5.5).

> $collect\left(factor\left(\dfrac{G2(p,\mathrm{sqrt}(1-p^2),y,\mathrm{mu},r)}{(1+p)\cdot(1-p)^2}\right),p,factor\right);$

$$16yr(\mu r+2y+2)p^3+4r\big(2\mu^2ry^2+2\mu ry^2+4\mu y^3-4\mu ry+4\mu y^2+4y^3+2ry-12y^2-r-16y\big)p^2-4y\big(4\mu^2r^2y-\mu r^2y+16\mu ry^2+2\mu r^2+16\mu ry-r^2y+4ry^2+8y^3+3r^2-10ry+16y^2-2r+8y\big)p+13\mu^2r^2y^2-12\mu r^2y^2+52\mu ry^3+4r^2\mu y+52\mu ry^2-r^2y^2-4ry^3+32y^4-28ry^2+64y^3+4r^2+24yr+32y^2 \tag{2}$$

>

> $G3 := (p,y,\mathrm{mu},r) \to 16yr(\mu r+2y+2)p^3+4r\big(4y^3+4y^2\mu-16y-r-12y^2+4y^3\mu$
$-4y\mu r+2yr+2y^2\mu r+2ry^2\mu^2\big)p^2-4y\big(3r^2+16y^2\mu r+4y^2r-10yr-yr^2$
$+2\mu r^2+16y^2+16y\mu r+4\mu^2r^2y-r^2\mu y+8y^3+8y-2r\big)p-12y^2\mu r^2+4r^2\mu y$
$-y^2r^2+32y^2+52y^2\mu r+52y^3\mu r+64y^3+24yr+13y^2\mu^2r^2-28y^2r+4r^2-4y^3r$
$+32y^4:$

> $collect\left(factor\left(\frac{G3\left(\frac{1}{2}\cdot(2-(\mathrm{mu}+1)\cdot y), y, \mathrm{mu}, r\right)}{8\cdot y^2}\right), y, factor\right);$

$$(2\mu+2)\,y^3+(\mu+1)\,(3\mu r+4)\,y^2+\left(r^2\mu^3+2+2\mu r-2r+\mu^2 r^2+2\mu+3\mu^2 r\right)y-r\,(\mu r-2\mu-1) \qquad (3)$$

This expression is the one written in equation (5.6). We maniplulate it to test its sign.

Use the fact that y^2+(2+mu*r)*y+1-r=0 to substitute and obtain the expression

> $collect\left(\left((2\mu+2)\,y+(\mu+1)\,(3\mu r+4)\right)\cdot(r-1-(2+\mathrm{mu}\cdot r)\cdot y)+\left(r^2\mu^3+2+2\mu r-2r+\mu^2 r^2+2\mu+3\mu^2 r\right)y-r\,(\mu r-2\mu-1), y, factor\right);$

$$-2\,(\mu+1)\,(2+\mu r)\,y^2+\left(-7\mu^2 r-2r^2\mu^3-8\mu-8-6\mu r-2\mu^2 r^2\right)y+3\mu^2 r^2-3\mu^2 r+3\mu r-4\mu+5r-4+2\mu r^2 \qquad (4)$$

Then again replace y^2 by r-1-(2+mu*r)*y to obtain

> $collect\left(-2\,(\mu+1)\,(2+\mu r)\cdot(r-1-(2+\mathrm{mu}\cdot r)\cdot y)+\left(-7\mu^2 r-2r^2\mu^3-8\mu-8-6\mu r-2\mu^2 r^2\right)y+3\mu^2 r^2-3\mu^2 r+3\mu r-4\mu+5r-4+2\mu r^2, y, factor\right);$

$$\mu r\,(\mu+2)\,y+r\left(\mu^2 r-\mu^2+\mu+1\right) \qquad (5)$$

Because r>1, then this expression is positive.