Sultan Qaboos University
Department of Mathematics and Statistics

Math2108 Summer 2009

Final Exam Time: 165 minutes

Name:. Section: . . . . . Number. . . . .

Important Instructions

• Make sure you write your name, number and section number on the exam paper and on the solution booklet.
• Solve all 10 questions. Make sure you show your complete, mathematically correct and neatly written solution.
• You are NOT allowed to share calculators or any other material during the test under any circumstances.
• Cellular phones are NOT allowed to be used as calculators or for any other purpose during the test.
• You should NOT ask the invigilator any questions about the exam.

Q1: Evaluate each one of the following integrals: **(4+5+5+6 points)**

$$(i)\ \int_3^{30} \frac{dx}{(x+2)^{\frac{3}{2}}} \qquad (ii)\ \int_0^2 x\tan^{-1}(x^2)\,dx$$

$$(iii)\ \int e^{5x}\sin^3(e^{5x})\cos(e^{5x})\,dx \qquad (iv)\ \int \frac{(x+2)(x-1)}{(x-2)(x^2+1)}\,dx$$

Q2: Solve each of (a) and (b). **(5+5 points)**

(a) Find the volume of the solid generated by revolving the region bounded by the curves $y=\ln(x), x=e$ and $y=0$ about the line $x=-1$.

(b) Determine whether the integral $\int_1^\infty \frac{dx}{(x+1)\ln(x+1)}$ is convergent or divergent.

Q3: Find the limit (if exists) in each of the following sequences: **(3+4+4 points)**

$$(i)\quad a_n=\frac{(-1)^n n}{3n+1} \qquad (ii)\quad b_n=\sqrt{(4n^2-n)}-2n \qquad (iii)\quad c_n=\sqrt{4-\frac{\sin(3^n)}{5^n}}.$$

Q4: Pick any 2 of the following series and determine whether they converge or diverge. If you solve more than 2, only the first two answered ones will be graded. **(5+5 points)**

$$(i)\ \sum_{k=1}^{\infty}\frac{3(-1)^k\,k}{\sqrt{k^2+1}} \qquad (ii)\ \sum_{k=1}^{\infty}\left(\frac{k}{2k+1}\right)^{2k} \qquad (iii)\ \sum_{k=1}^{\infty}\frac{(k^2)(k!)}{(2k)!}$$

Q5: Solve each of (a) and (b) **(4 +4 points)**

(a) Find the sum $\sum_{k=1}^{\infty}\frac{2}{k(k+1)}$

(b) Show that $\sum_{k=0}^{\infty}\frac{(-1)^k}{3^k}\cos^k(3x)$ is a geometric series, then find the sum if it exists.

Q6: Answer each of (a) and (b). **(5 +5 points)**

(a) Consider $f(x)=\ln(x)$.

(i) Construct the Taylor series for $f(x)$ about $c=1$.

(ii) Find the Taylor series for $g(x)=(x-1)f(x)$ about $c=1$.

(b) Find the Interval and Radius of Convergence for the power series $\sum_{k=0}^{\infty}\frac{(x-1)^k}{2^k}$.

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Q7: Answer each of (a) and (b).

(a) Given that the Maclaurin series for $f(x) = \frac{x}{1+x^2}$ is

$$x - x^3 + x^5 + ... + (-1)^{k+1}x^{2k-1} + \cdots, \qquad -1 < x < 1.$$

(i) Find the Maclaurin series for $\frac{1}{2}\ln(1+x^2)$. Use sum notation. **(5 points)**

(ii) Evaluate the sum $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k(2)^{2k}}$. **(3 points)**

(b) Given that **(3 points)**

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \qquad |x| < 1.$$

Find the Taylor series for $\frac{1}{1+4x^2}$ about $c = 0$.

Q8: Circle the correct answer. **(2.5 points each)**

(a) One of the following series is convergent:

$(i)\ \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ $\quad (ii)\ \sum_{k=1}^{\infty} 1$ $\quad (iii)\ \sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ $\quad (iv)\ \sum_{k=1}^{\infty} (-1)^k$

(b) One of the following series is divergent:

$(i)\ \sum_{k=1}^{\infty} \left(\frac{-1}{4}\right)^k$ $\quad (ii)\ \sum_{k=1}^{\infty} \frac{1}{k}$ $\quad (iii)\ \sum_{k=1}^{\infty} \frac{x^k}{k!}$ $\quad (iv)\ \sum_{k=1}^{\infty} \frac{\sin(k\pi)}{k\pi}$

(c) One of the following series is absolutely convergent:

$(i)\ \sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{k^3+5}$ $\quad (ii)\ \sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ $\quad (iii)\ \sum_{k=1}^{\infty} (-\sqrt{3})^k$ $\quad (iv)\ \sum_{k=1}^{\infty} \frac{\cos(k\pi)}{k}$

(d) The polar point $(r, \theta) = (-\sqrt{2}, \frac{\pi}{4})$ has the rectangular representation $(x, y) =$
(i) $(1, 1)$ (ii) $(-1, -1)$ (iii) $(-1, 1)$ (iv) $(1, -1)$

Q9: Match each equation with the correct answer in the right column. **(1 point each)**

#	The polar equation	The graph
(i)	$r = -3$	a line
(ii)	$\theta = \frac{3\pi}{5}$	a circle
(iii)	$r = \sin(\theta)$	a cardioid
(iv)	$1 = \frac{2\sin(\theta)}{\sin(\theta)+\cos(\theta)}$	a point
(v)	$r = \sin(\theta) + \cos(\theta)$	a parabola

Q10: State whether True or False. No need for justification. **(1 point each)**

(i) A non-power series and its derivative have the same radius of convergence.

(ii) If $\sum_{k=1}^{\infty} a_k$ converges, then $\lim_{k\to\infty} a_k = 0$.

(iii) If a series is convergent absolutely, then it is convergent.

(iv) If both $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ diverge, then $\sum_{k=1}^{\infty}(a_k - b_k)$ diverges.

(v) If a series is convergent, then it is convergent absolutely.

(vi) If a series is conditionally convergent, then it is convergent.

(vii) Each rectangular point (x, y) has a unique (only one) polar representation (r, θ).

Good Luck