A 200$ Problem for Calculus Students*

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Given constant parameters $b > 1$ and $0 < h < (\sqrt{b}-1)^2$. Define $f(t) = \frac{1}{b}(1+t)(h+t)$, $\bar{x}_2 := \max\{t : \ t = f(t)\}$ and $g(t) = \frac{1}{b}(\bar{x}_2 + h)(1+t)$. Prove that if x, y satisfy $x \geq f\left(\frac{b+h}{\bar{x}_2}\right)$ and $\min\{\bar{x}_2, g(x)\} \leq y \leq \max\{\bar{x}_2, g(x)\}$, then

$$\frac{(y+h+b)(f(x)-y)(g(x)-y)}{y(1+x)} \geq \frac{(x+h+b)(x-f(y))(y-\bar{x}_2)}{bx-(1-h)(y+h)}.$$

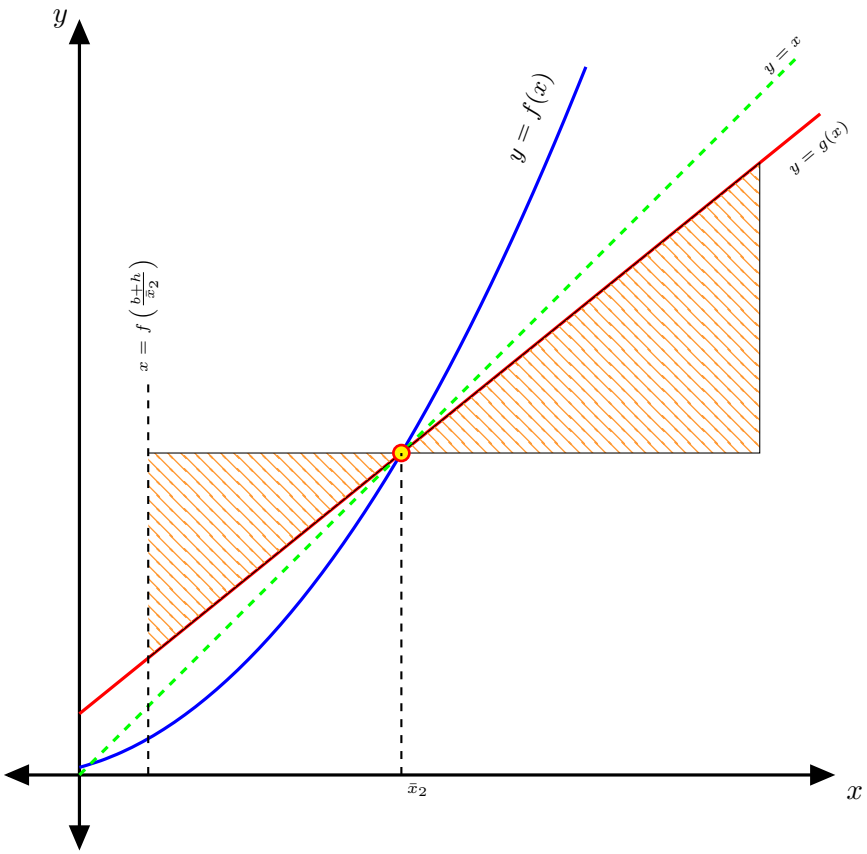

Figure 1: The shaded region represents the values of (x, y) that satisfy the inequality. The scale on the axes is intentionally missing because the graphs represent a general sketch.

*A gift of $200 and a dinner will be given to the first person who sends me a complete solution. The first correct and complete solution I get will be posted in my website at www.alsharawi.info