Periodic Structures in Iterated Maps

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Let $f(x)$ be a continuous function defined on a closed interval I, and let $x_0 \in I$. Can we characterize the asymptotic behaviour of the sequence

$$x_0,\ f(x_0), f \circ f(x_0),\ f \circ f \circ f(x_0),\ f \circ f \circ f \circ f(x_0), \cdots ? \tag{1}$$

This sequence can be viewed as the solution of a difference equation in the form $x(n+1) = f(x(n))$, which is a discrete analog of the differential equation $x'(t) = f(x(t))$.

On the other hand, a discrete analog of the non-autonomous differential equation $x'(t) = f(t, x(t))$ is given by the difference equation $x(n+1) = f(n, x(n))$, i.e., given a sequence of continuous functions $f_0, f_1, f_2, ...$ defined on a closed interval I, we investigate the alternating system

$$x_0,\ f_0(x_0), f_1 \circ f_0(x_0),\ f_2 \circ f_1 \circ f_0(x_0),\ f_3 \circ f_2 \circ f_1 \circ f_0(x_0), \cdots \tag{2}$$

In this seminar, we use tools from analysis and graph theory to characterize periodic structures obtained in orbits (1) and (2). The presentation will be given assuming the audience is not specialized in difference equations but has a diverse background in mathematics.

References:

[1] Z. AlSharawi, J. Canovas, A. Linero, Folding and unfolding in periodic difference equations, Pre-print, www.alsharawi.info

[2] Z. AlSharawi and A. Al-Salman, A new characterization of periodic oscillations in periodic difference equations, *Chaos, Solitons & Fractals,* **44** (2011) 921-928.

[3] Z. AlSharawi, Periodic orbits in periodic discrete dynamics, *Computers and Mathematics with Applications,* **56** (2008) 1966–1974.

[4] Z. AlSharawi, J. Angelos, S. Elaydi, An extension of Sharkovsky's theorem to periodic difference equations, *Journal of Mathematical Analysis and Applications* **316** (2006) 128–141.